Bifurcation Theory, a.y. 2013/14

Homework #1: Perturbation Methods Part1

November 12, 2013

Exercise 1:

Find an asymptotic expansion for $\varepsilon \to 0$, to within an error $O\left(\varepsilon^3\right)$, for the roots of the following equation:

$$\sin x = \varepsilon$$

Exercise 2:

Solve the following set of algebraic equations when $(x, y, p) = O(\varepsilon)$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} y^2 + x^2 y \\ xy \end{pmatrix} = p \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Exercise 3:

Find asymptotic expansions for the eigenvalues of the following problem:

$$\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \varepsilon \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

[Solution: $\lambda_1 = -1 - \varepsilon - \varepsilon^2$, $\lambda_{2,3} = \mp\sqrt{\varepsilon} + \frac{1}{2}\varepsilon \pm \frac{1}{8}\varepsilon^{3/2} + \ldots$]

Exercise 4:

By using a perturbation method, find the branch point and plot the bifurcation diagram $\mu = f(x, y)$ for the following statical system:

$$\begin{bmatrix} \mu & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sin x - x - 2xy \\ \cos x - 1 \end{pmatrix}$$